

Artificial Intelligence and Automation

Unit 2: Artificial Neural Networks ANN

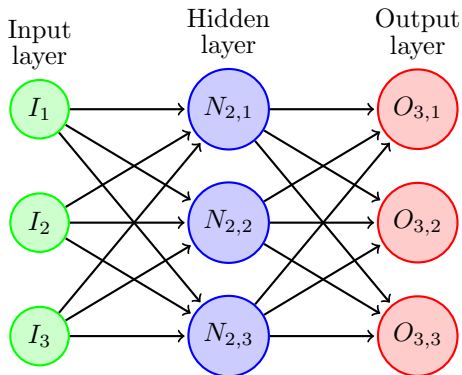
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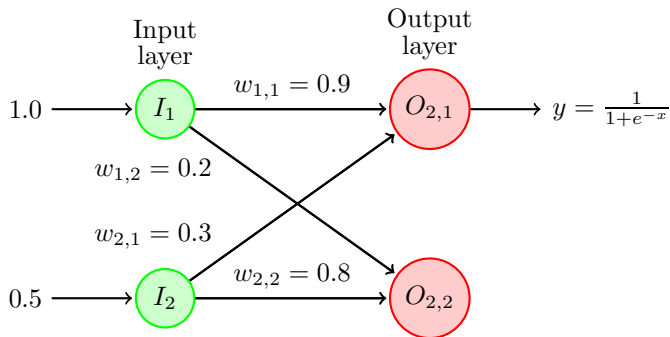
Introduction

- ▶ Inputs and outputs
- ▶ Neuron and its activation function
- ▶ Weights and bias



Signals in the Neural Network

Considers the next example and compute first the output $O_{2,1}$

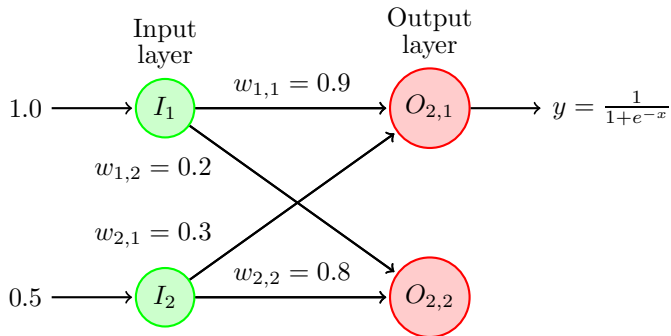


$$x = I_1 \cdot w_{1,1} + I_2 \cdot w_{2,1}$$



Signals in the Neural Network

Considers the next example and compute first the output $O_{2,1}$



$$x = I_1 \cdot w_{1,1} + I_2 \cdot w_{2,1}$$

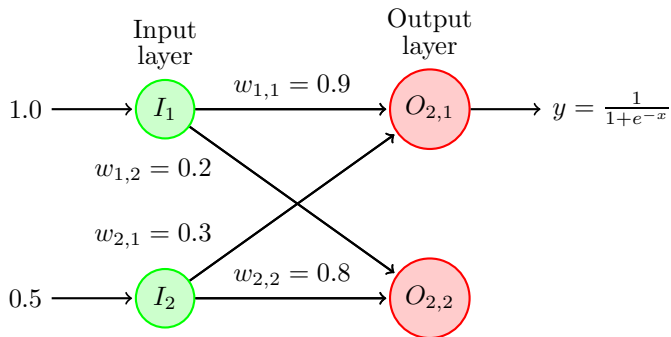
$$x = 1.0 \cdot 0.9 + 0.5 \cdot 0.3$$

$$x = 1.05$$



Signals in the Neural Network

Considers the next example and compute first the output $O_{2,1}$



$$x = I_1 \cdot w_{1,1} + I_2 \cdot w_{2,1}$$

$$x = 1.0 \cdot 0.9 + 0.5 \cdot 0.3$$

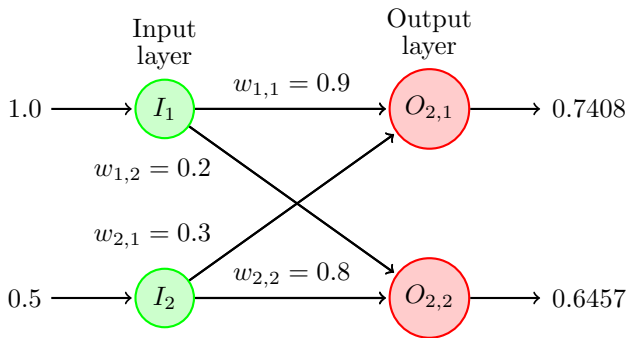
$$x = 1.05$$

$$y = \frac{1}{1 + 0.3499} = 0.7407$$



Signals in the Neural Network

Results



Matrix Multiplication

Then, W is the matrix of weights, I is the matrix of inputs, and X is the resulting matrix of combined moderated signals into layer 2.

$$W \cdot I = X \quad (1)$$

$$\begin{bmatrix} w_{1,1} & w_{2,1} \\ w_{1,2} & w_{2,2} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} (I_1 w_{1,1}) + (I_2 w_{2,1}) \\ (I_1 w_{1,2}) + (I_2 w_{2,2}) \end{bmatrix} \quad (2)$$

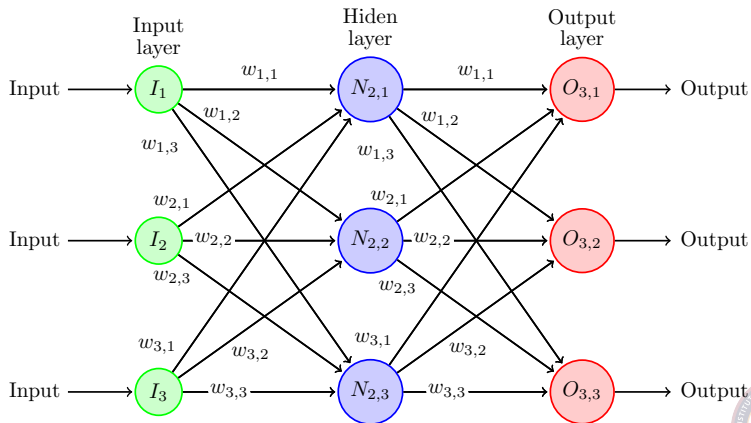
Finally, the output of the layer is:

$$O = \text{sigmoid}(X)$$



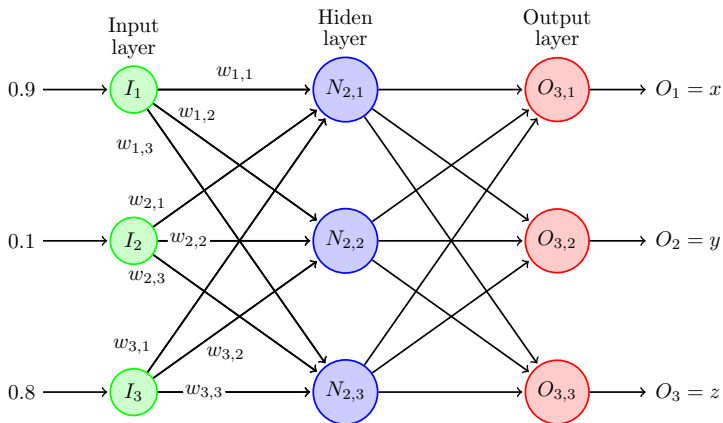
A Three Layer Matrix Multiplication

Terminology



Three layer example

Input-Hidden Layer



$$w_{11} = 0.9, \quad w_{12} = 0.2, \quad w_{13} = 0.1,$$

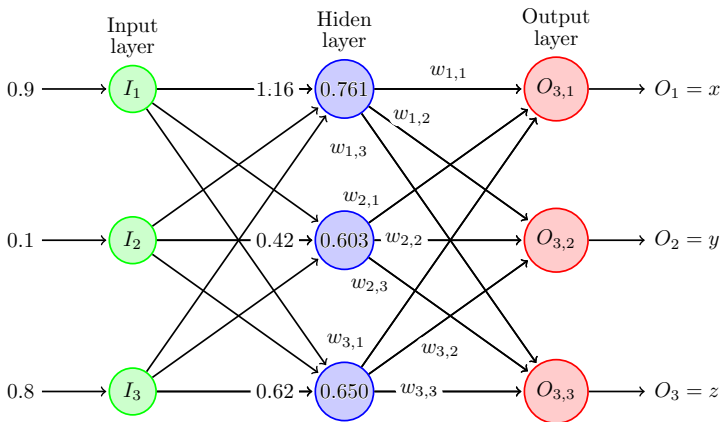
$$w_{21} = 0.3, \quad w_{22} = 0.8, \quad w_{23} = 0.5,$$

$$w_{31} = 0.4, \quad w_{32} = 0.2, \quad w_{33} = 0.6,$$



Three layer example

Hidden-Output Layer



$$w_{11} = 0.3, \quad w_{12} = 0.6, \quad w_{13} = 0.8,$$

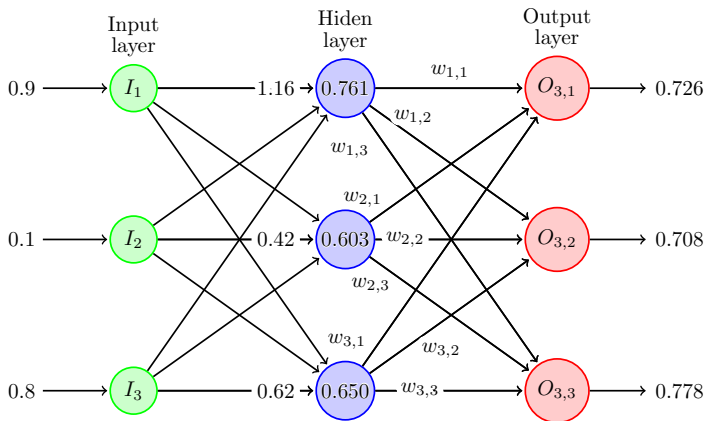
$$w_{21} = 0.7, \quad w_{22} = 0.5, \quad w_{23} = 0.2,$$

$$w_{31} = 0.5, \quad w_{32} = 0.2, \quad w_{33} = 0.9,$$

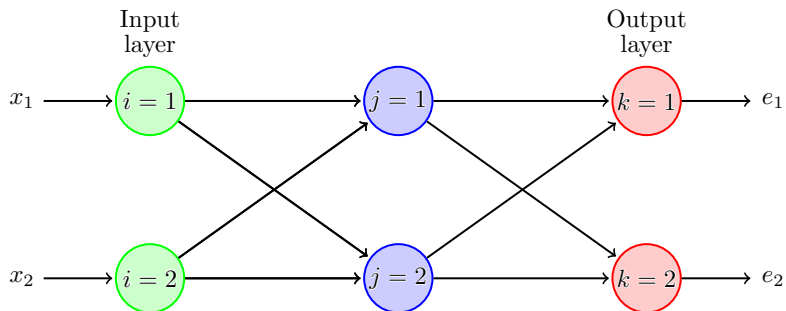


Three layer example

Resulting Output



The Error

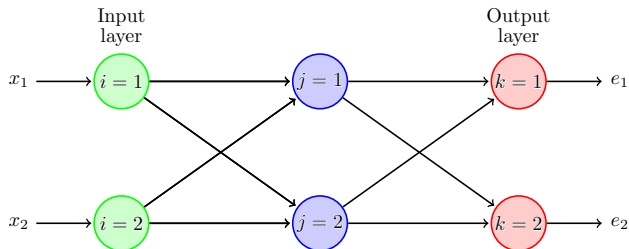


$$e_k = t_k - o_k$$



The Error

Backpropagation



$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad (5)$$

$$e_h = \begin{bmatrix} \frac{w_{11}}{w_{11}+w_{21}} & \frac{w_{12}}{w_{12}+w_{22}} \\ \frac{w_{21}}{w_{21}+w_{11}} & \frac{w_{22}}{w_{22}+w_{12}} \end{bmatrix} \quad (6)$$



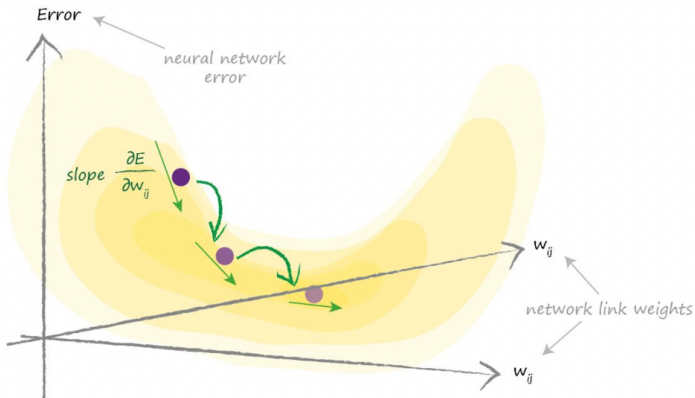
$$e_h = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad (7)$$

$$e_h = W_{ho}^T \cdot e_{out} \quad (8)$$



The Error

Gradient concept



$$\frac{\partial E}{\partial w_{jk}} = \frac{\partial}{\partial w_{jk}} \sum_n (t_n - o_n)^2$$



The Error

Gradient formula

$$\frac{\partial E}{\partial w_{jk}} = \frac{\partial}{\partial w_{jk}} \sum_n (t_n - o_n)^2$$

- ▶ o_n only depends on the links connected to it
- ▶ o_k depends on w_{jk}

$$\frac{\partial E}{\partial w_{jk}} = \frac{\partial}{\partial w_{jk}} (t_k - o_k)^2$$

(10)



The Error

Gradient formula

$$\frac{\partial E}{\partial w_{jk}} = \frac{\partial}{\partial w_{jk}} (t_k - o_k)^2$$

- ▶ t_k is constant
- ▶ o_k depends on w_{jk}



The Error

Gradient formula: Chain Rule

$$\frac{\partial E}{\partial w_{jk}} = \frac{\partial E}{\partial o_k} \cdot \frac{\partial o_k}{\partial w_{jk}}$$



The Error

Gradient formula: Chain Rule

$$\frac{\partial E}{\partial w_{jk}} = \frac{\partial E}{\partial o_k} \cdot \frac{\partial o_k}{\partial w_{jk}}$$

$$\frac{\partial E}{\partial w_{jk}} = -2(t_k - o_k) \cdot \frac{\partial}{\partial w_{jk}} \sigma \left(\sum_j w_{jk} o_j \right)$$

o_j is the output of the previous hidden layer node; the input of the current layer!!



The Error

Gradient formula: Chain Rule

$$\frac{\partial}{\partial x} \sigma(x) = \sigma(x) (1 - \sigma(x))$$



The Error

Gradient formula: Chain Rule

$$\frac{\partial}{\partial x} \sigma(x) = \sigma(x) (1 - \sigma(x))$$

$$\frac{\partial E}{\partial w_{jk}} = -2(t_k - o_k) \cdot \sigma \left(\sum_j w_{jk} o_j \right) \left(1 - \sigma \left(\sum_j w_{jk} o_j \right) \right) o_j$$



The Error

Updating weights

$$\frac{\partial E}{\partial w_{jk}} = -e_j \cdot \sigma \left(\sum_i w_{ij} o_i \right) \left(1 - \sigma \left(\sum_i w_{ij} o_i \right) \right) o_i \quad (11)$$

- ▶ e_j is the error at the output
- ▶ The σ refers to the previous layers; the hidden node j
- ▶ o_i is the output of the first layers of nodes

$$w_{jk} = w_{jk} - \alpha \frac{\partial E}{\partial w_{jk}}$$



The Error

Updating weights

$$\begin{bmatrix} \Delta w_{11} & \Delta w_{12} & \Delta w_{13} & \cdots \\ \Delta w_{21} & \Delta w_{22} & \Delta w_{23} & \cdots \\ \Delta w_{31} & \Delta w_{32} & \Delta w_{jk} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} = \begin{bmatrix} E_1 \sigma_1 (1 - \sigma_1) \\ E_2 \sigma_2 (1 - \sigma_2) \\ E_k \sigma_k (1 - \sigma_k) \\ \cdots \end{bmatrix} [o_1 \quad o_2 \quad o_j \cdots] \quad (13)$$

- ▶ k values from next layer
- ▶ j values from previous layer



References

-  Rashid, Tariq. Make your own neural network. CreateSpace Independent Publishing Platform, 2016.

